Journal of Engineering Physics and Thermophysics, Vol. 75, No. 1, 2002

PHASE DIAGRAM OF A DISPERSE SYSTEM WITH ASCENDING GAS FLOW

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UDC 533.6

The phase diagram of an infiltrated disperse medium which includes a circulating fluidized bed in addition to fixed and fluidized beds and vertical pneumatic transport has been constructed. The expression to calculate the most important characteristic of a flow system, i.e., the transport velocity, has been obtained. The definition of a circulating fluidized bed has been given.

As is well known, disperse infiltrated beds with ascending gas flow (fixed bed (FixB), fluidized bed (FlB), circulating fluidized bed (CFlB), and vertical pneumatic transport (VPT)) are widely used in industry. The similarity of the processes of transfer in these systems makes it possible to combine them into one series and consider them in the general theoretical context. In [1], a unified system of dimensionless parameters which describes the similarity of the processes of transfer in disperse beds with ascending gas flow has been obtained and its capacity for rationally generalizing experimental data has been shown. As is well known, a circulating fluidized bed, unlike other disperse systems, has been used only comparatively recently and is still incompletely understood. Being the intermediate link between a fluidized bed and vertical pneumatic transport, it has absorbed many features of these systems and is capable of operating in a very wide range of velocities of a gas and flow rates of a solid material. Because of this, the difficulties arising on an attempted mathematical description of a circulating fluidized bed seem to be insurmountable at present. As has been noted in [2], there is not even a unified definition of a circulating fluidized bed, and its boundaries in the phase diagram have not been established. In this connection, in the present work we have made an attempt to describe a circulating fluidized bed within the framework of a unified phase diagram of a disperse system with ascending gas flow and to establish the boundaries of its existence.

It is well known that the distinctive features of evolution of infiltrated disperse systems may be conveniently analyzed using a phase diagram which represents the plot of the pressure difference per unit height of the bed versus the velocity of the gas flow. Such a diagram for the systems fixed bed — fluidized bed — vertical pneumatic transport was proposed for the first time, apparently, by Zenz [3]. For the circulating fluidized bed to be included into this diagram, to be specific, we take the following parameters of the disperse systems: air at room temperature and atmospheric pressure as the fluidizing agent, solid particles with d = 0.32 mm and ρ_s = 2600 kg/m^3, bed height H_0 = 0.5 m (fixed bed and fluidized bed) and H = 13.5 m (circulating fluidized bed and vertical pneumatic transport), diameter of the apparatus D = 0.5 m, and value of the circulation particle flux J_s = 50 and 100 (kg/m^2·sec) (circulating fluidized bed and vertical pneumatic transport).

Let us consider the formulas to calculate the dependence $\Delta p/\Delta h = f(u)$ in different disperse layers.

Fixed Blown-Through Bed. The quantity $\Delta p/\Delta h$ is calculated from the known Ergun formula [4]

$$\frac{\Delta p}{\Delta h} = 150\,\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\,\frac{\mu_f u}{d^2} + 1.75\,\frac{1-\varepsilon_0}{\varepsilon_0^3}\,\frac{\rho_f u^2}{d}\,. \tag{1}$$

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 1, pp. 117–121, January–February, 2002. Original article submitted February 20, 2001.

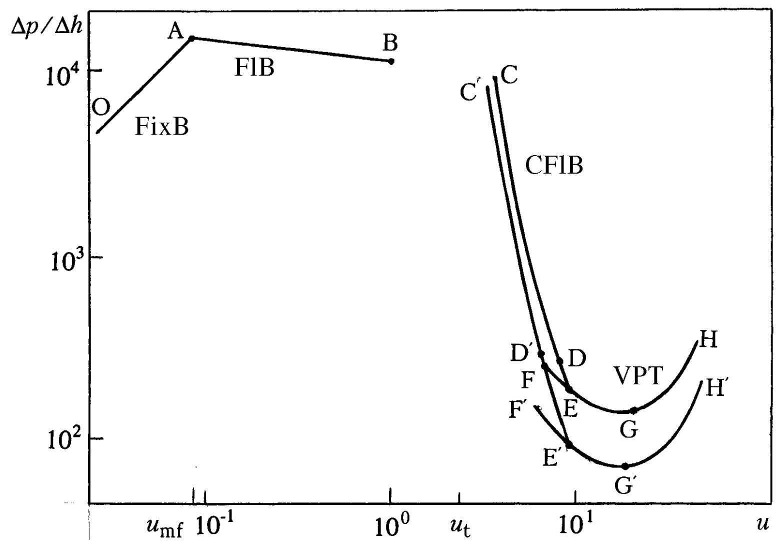

Fig. 1. Phase diagram of a disperse medium with ascending gas flow: OA, fixed bed; AB, fluidized bed; CDE and C′D′E′, circulating fluidized bed for J_s = 100 and 50 kg/(m^2·sec); FGH and F′G′H′, vertical pneumatic transport for J_s = 100 and 50 kg/(m^2·sec); u_{mf} = 0.086 m/sec (point A); u_{sl} = 0.96 m/sec (point B); u_t = 2.27 m/sec; u_d = 6.3 and 5.7 m/sec for J_s = 100 and 50 kg/(m^2·sec) (points F and F′); u_* = 8.15 and 6.7 m/sec for J_s = 100 and 50 kg/(m^2·sec) (points D and D′); u_{tr} = 9.27 m/sec (points E and E′); u_{opt} = 23.5 and 20.7 m/sec for J_s = 100 and 50 kg/(m^2·sec) (points G and G′. $\Delta p/\Delta h$, P/m; u, m/sec.

Point A on the phase diagram (Fig. 1) corresponds to the moment of suspension of the bed

$$\left.\frac{\Delta p}{\Delta h}\right|_{u=u_{mf}} = g\,(\rho_s - \rho_f)\,(1 - \varepsilon_{mf})\,. \tag{2}$$

Combination (1) for $\varepsilon_0 = \varepsilon_{mf}$ and $u = u_{mf}$ and Eq. (2) determine the rate of the beginning of fluidization u_{mf} [5].

Fluidized Bed. By virtue of the two-phase structure of the bed [6], its total resistance remains approximately constant; therefore, $\Delta p/\Delta h$ decreases in accordance with the expansion of the bed $H(u)$:

$$\frac{\Delta p}{\Delta h} = \frac{g\,(\rho_s - \rho_f)\,(1 - \varepsilon_{mf})\,H_{mf}}{H\,(u)}\,. \tag{3}$$

We give one of the most substantiated formulas for calculation of the expansion of the bed [7]:

$$H\,(u) = H_{mf}\left(1 + 0.7\left(\frac{H_{mf}}{D}\right)^{1/2} \mathrm{Fr}^{1/3}\right). \tag{4}$$

Portion AB on the phase diagram corresponds to the calculation according to (3) with account for (4). At point B, the piston regime sets in, in which the diameter of a gas bubble becomes approximately equal to the diameter of the apparatus: $D_b\big|_{h=H_{mf}} = D$. This condition enables us to determine the rate u_{sl}, i.e., the coordinate of point B:

$$u_{sl} \approx u_{mf} + 0.395 \frac{g^{1/2} D^{3/2}}{H_{mf}} . \qquad (5)$$

In (5), we have used the known formula for the diameter of a gas bubble [8]:

$$D_b = \frac{D_h}{0.7} = 1.86h \, \mathrm{Fr}_h^{1/3} . \qquad (6)$$

When the piston regime sets in curve AB terminates since, because of the strong variations of the upper boundary of the bed, it becomes impossible to quantitatively analyze the dependence $\Delta p/\Delta h = f(u)$. With further increase in the rate u/u_{sl} the intense removal of particles from the nonflow fluidized bed begins. By convention, only one particle is left in the system for $u = u_t$. This state corresponds to the free-fall velocity of a single particle u_t. Many formulas have been proposed to calculate this quantity; the most convenient of them is the Todes formula [5].

Vertical Pneumatic Transport. We consider it after a fluidized bed as a system that is more completely understood than a circulating fluidized bed and not in order of increasing rate of filtration of the gas. Unlike a fixed bed and a fluidized bed, here we have an important additional parameter characteristic of a flow system, i.e., the circulation particle flux J_s. The pressure gradient is calculated from the formula [9]

$$\frac{\Delta p}{\Delta h} = \frac{J_s g}{u - u_t} + \frac{1.36}{D} J_s^{-1/2} \frac{\rho_f u^2}{2} + \frac{J_s (u - u_t)}{2H} , \qquad (7)$$

which yields the family of curves (for different values of J_s) on the phase diagram. We note that, as compared to [9], Eq. (7) contains the additional term $J_s(u - u_t)/2H$, which takes into account the influence of the acceleration of particles. The lower boundary of existence of the vertical pneumatic transport (points F and F$'$) correspond to the known descent velocity [10]. We give the recent and most reliable formulas for determination of this quantity [9]:

$$\frac{u_d - u_t}{u_t} = 0.11 B^{0.5} , \quad d \le 0.28 \text{ mm} ; \qquad (8)$$

$$\frac{u_d - u_t}{u_t} = 0.02 J_s^* , \quad 0.586 \le d \le 1.67 \text{ mm} . \qquad (9)$$

At points G and G$'$, the curves $\Delta p/\Delta h = f(u)$ have a minimum whose position is determined from the following dependences [9]:

$$u_{opt} = u_t (4/3 + 0.65 B^{0.8}) , \quad B \le 1/3 , \qquad (10)$$

$$u_{opt} = u_t (1 + B^{0.4}) , \quad B > 1/3 . \qquad (11)$$

The region to the right of points G and G$'$, i.e., GH and G$'$H$'$, where the contribution of the forces of friction of the two-phase medium against the surface of the riser begins to have an effect, is characterized by a rapid growth in the pressure difference with increase in the velocity of the gas.

Circulating Fluidized Bed. Curves CE and C$'$E$'$ for different values of J_s were constructed according to the formula

$$\frac{\Delta p}{\Delta h} = 0.6 \frac{J_s}{H} (u - u_t) \, \mathrm{Fr}_t^{-1.68} , \tag{12}$$

obtained on the basis of the empirical dependence for calculation of the resistance of the riser [11]:

$$\Delta p = 0.6 \, J_s \, (u - u_t) \, \mathrm{Fr}_t^{-1.68} . \tag{13}$$

We note that because of the inhomogeneous structure of the circulating fluidized bed [2] the real values of the pressure gradient $\Delta p/\Delta h$ will substantially vary over the riser height and expression (12) should be considered as the averaged value of $\Delta p/\Delta h$ making it possible to present the circulating fluidized bed on the phase diagram. At present, there are no definite recommendations on the position of points C and C$'$ that correspond to the beginning of the region of the circulating fluidized bed. It is clear that these points will be near u_t and their specific location depends on J_s and is determined by the suspension-carrying capacity of the flux. The positions of the last right-hand points E and E$'$ are, apparently, determined as the points of intersection of curves FGH and F$'$G$'$H$'$ for the pneumatic transport, i.e., from the condition

$$\left.\frac{\Delta p}{\Delta h}\right|_{\mathrm{CFlB}} = \left.\frac{\Delta p}{\Delta h}\right|_{\mathrm{VPT}} , \tag{14}$$

which, with account for (7) and (12), will take the form

$$\frac{J_s g}{u - u_t} + 1.36 \, \bar{J}_s^{1/2} \frac{\rho_f u^2}{2D} + \frac{J_s \, (u - u_t)}{2H} = 0.6 \frac{J_s}{H} (u - u_t) \, \mathrm{Fr}_t^{-1.68} . \tag{15}$$

As the evaluations show, for the velocities $u < u_{opt}$ one can disregard the influence of friction against the riser walls (second term on the left-hand side of (15)). Equation (15) is simplified

$$0.6 \, \mathrm{Fr}_t^{-0.68} - 0.5 \, \mathrm{Fr}_t - 1 = 0 \tag{16}$$

and yields the following expression to find the sought rate:

$$(\mathrm{Fr}_t)_{tr} = \frac{(u_{tr} - u_t)^2}{gH} = 0.37 . \tag{17}$$

To elucidate the physical conditions that correspond to $u = u_{tr}$ it is convenient to analyze the expenditure of the gas power in the circulating fluidized bed and in the vertical pneumatic transport. As follows from (7), the excess power of the gas $\Delta p(u - u_t)$ in the case of pneumatic transport is equal to

$$N_{\mathrm{VPT}} = \Delta p \, (u - u_t) = J_s g H + \frac{J_s \, (u - u_t)^2}{2} + \frac{1.36}{D} \bar{J}_s^{1/2} \frac{\rho_f u^2}{2} (u - u_t) \, H . \tag{18}$$

From (18) it is seen that the excess power of the gas (fan) is expended on: (a) lifting the particles in the gravity field, (b) accelerating them from 0 to $u - u_t$ on the acceleration portion, and (c) overcoming the forces of friction against the channel walls. The analogous expression can, apparently, be written for a circulating fluidized bed, where one should also take into account another category of expenditure of the gas energy (this category is characteristic precisely of a circulating fluidized bed), i.e., expenditure of energy on sustaining the internal circulation of particles ($N_{\mathrm{int.c}}$):

$$N_{\mathrm{CFlB}} = J_s gH + \frac{J_s (u - u_t)^2}{2} + N_{\mathrm{int.c}} + N_{\mathrm{fr}} \,. \tag{19}$$

We note that for a circulating fluidized bed (unlike vertical pneumatic transport) a specific expression to calculate the expenditure of energy on friction against the riser walls remains to be obtained, and this term is written only in general form. It is clear that at the point $u = u_{\mathrm{tr}}$ $N_{\mathrm{CFlB}} = N_{\mathrm{VPT}}$, and Eqs. (18) and (19) yield

$$\frac{1.36}{D} H \bar{J}_s^{1/2} \frac{\rho_f u^2}{2} (u - u_t) = N_{\mathrm{int.c}} + N_{\mathrm{fr}} \,. \tag{20}$$

Using the natural assumption that for $u = u_{\mathrm{tr}}$ the expenditure of energy on friction against the riser walls is the same in the two systems, from (2) we have the condition which corresponds to $u = u_{\mathrm{tr}}$:

$$N_{\mathrm{int.c}} = 0 \,, \tag{21}$$

i.e., at the point $u = u_{\mathrm{tr}}$, the internal circulation of particles in the circulating fluidized bed disappears and the difference between the circulating fluidized bed and vertical pneumatic transport exists no longer, in essence. Consequently, the velocity $u = u_{\mathrm{tr}}$ can be identified with the transport velocity, to calculate which Eq. (17) yields the simple formula

$$u_{\mathrm{tr}} = u_t + 0.61 \sqrt{gH} \,. \tag{22}$$

Disregarding the contribution of friction against the riser walls (just as in the case of vertical pneumatic transport, this is allowable for $u < u_{\mathrm{opt}}$), from (13) and (19) we obtain an expression for calculation of the fan power expended on sustaining the internal circulation of particles:

$$N_{\mathrm{int.c}} = J_s gH \,(0.6\, \mathrm{Fr}_t^{-0.68} - 1) - \frac{J_s (u - u_t)^2}{2} \,. \tag{23}$$

In connection with the foregoing, we can give the following definition of a circulating fluidized bed: a circulating fluidized bed is a *flow two-phase system with a marked internal circulation of particles.*

Points D and D′ on the phase diagram characterize another important aspect in the evolution of a circulating fluidized bed with smooth increase in the gas velocity, i.e., the disappearance of a fluidized bed that exists at the gas-distributor grid [12]. The corresponding velocity (u_*) can be evaluated from the following considerations. In [11], the author obtained a formula to calculate the height of the fluidized bed at the gas distributor in a circulating fluidized bed:

$$\frac{z}{H} = 1.25\, \mathrm{Fr}_t^{-0.8} \bar{J}_s^{1.1} \,. \tag{24}$$

By assuming that the fluidized bed virtually disappears, attaining a certain small critical value of $z^* = AH$, from (24) we obtain for u_*

$$\frac{u_* - u_t}{u_t} = \frac{1.25}{A} \tilde{J}_s^{*\,0.4}\, \mathrm{Fr}_H^{-0.3} \,. \tag{25}$$

Processing of the experimental data on u_* which are available in the literature [12, 13] (Fig. 2), enabled us to find $A = 0.016$. The resultant dependence for calculation of the velocity u_* at which the fluidized bed at the gas distributor disappears has the form

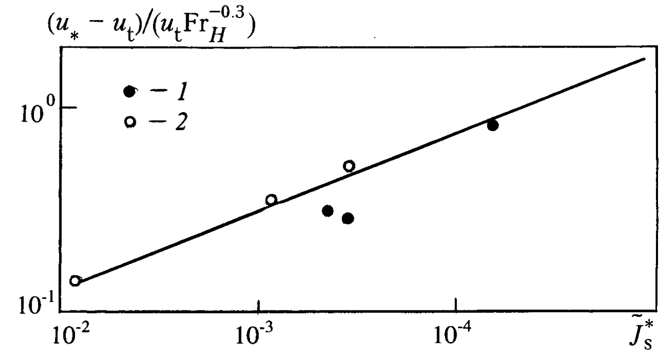

Fig. 2. Velocity for which the fluidized bed disappears: 1) [13] and H = 8.5 m; 2) [12] and H = 13.5 m. The solid line denotes calculation according to (26).

$$\frac{u_* - u_t}{u_t} = 5.0\, \tilde{J}_s^{*\,0.4}\, \mathrm{Fr}_H^{-0.3} . \tag{26}$$

Formula (26) has been checked in the following range of variation of experimental conditions: $0.15 \le d \le 0.46$ mm, $1.1 \le J_s \le 45$ kg/(m^2·sec), H = 8.5 and 13.5 m, and ρ_s = 2600 kg/m^3.

Let us consider the issue of the hydrodynamic conditions of descent of the particles (formation of the contour of their internal circulation) in the case of a smooth decrease in the gas velocity in the flow system. As is well known [2], the internal circulation of particles in a circulating fluidized bed is formed by the clusters of particles descending at the riser walls and by single particles ascending at the center. Based on such a structure of circulation flows, the formation of the internal circulation of particles in a pneumatic-transport system with gradual decrease in the filtration rate of the gas seems to be as follows. Since the concentration of the particles under conditions of vertical pneumatic transport is expressed [9] as

$$1 - \varepsilon = \bar{J}_s = \frac{J_s}{\rho_s\,(u - u_t)} , \tag{27}$$

it follows that, as the gas velocity decreases (for a constant J_s), the concentration of particles increases. By virtue of the existing nonuniformity of the gas flow in the horizontal section of the riser [2], the concentration of particles at the walls will be substantially (2 to 3 times) higher than the average concentration. With the universally present local turbulent pulsations of the gas velocity and hence the concentration of particles, the probability of appearance of bluff groups of particles that are unable to be held by the gas flow increases under these conditions. These macroformations begin to fall down, producing the lowering circulation motion of the particles.

Based on such concepts, it is easy to evaluate the minimum size of clusters formed at gas velocities which are close to u_{tr}. To calculate the vertical dimension of a cluster a simple formula was obtained [14] which relates this quantity to the concentration of particles:

$$\frac{L}{H} = 0.024 \sqrt{1 - \varepsilon} . \tag{28}$$

When $u = u_{tr}$ Eqs. (27) and (28) yield for L_{min}

$$\frac{L_{min}}{H} = 0.024 \sqrt{\frac{J_s}{\rho_s\,(u_{tr} - u_t)}} . \tag{29}$$

With account for formula (22) we finally obtain

$$\frac{L_{\min}}{H} = 0.031 \sqrt{\frac{J_s}{\rho_s \sqrt{gH}}} \ . \tag{30}$$

Let us make more specific the calculation for the conditions of [15], where the vertical dimensions of the clusters in a circulating fluidized bed 0.15 m in diameter and 11 m high have been measured with a capacitive pickup. For J_s = 45 kg/(m^2·sec), d = 0.251 mm, and ρ_s = 2600 kg/m^3, Eq. (30) yields $L_{\min} \approx 0.013$ m, which corresponds to $L_{\min}/d \approx 52$. The obtained value of $L_{\min}$ is close to the minimum value of L measured in [15]: $L_{\min} \approx 0.018$ m.

It is of interest to evaluate the values of the average concentrations of particles under the conditions where clusters begin to form, i.e., when $u = u_{tr}$. From (22) and (27) it follows that

$$(1 - \varepsilon)_{\min} = \frac{J_s}{\rho_s (u_{tr} - u_t)} = 1.64 \frac{J_s}{\rho_s \sqrt{gH}} \ . \tag{31}$$

Hence for the operating conditions mentioned above [15] we have $(1 - \varepsilon)_{\min} \approx 0.0027$, which corresponds to $\varepsilon_{\max} = 0.9973$. For the wall values of the concentrations of particles we obtain $(1 - \varepsilon)_w = (2 - 3(1 - \varepsilon)_{\min} \approx 0.0054–0.0081$.

As is seen from the constructed phase diagram (Fig. 1), the values of the transport velocity u_{tr} and of the descent velocity u_d differ rather significantly. The reasons are not quite clear yet and, in our opinion, have to do with the distinctive features of the real geometry of apparatuses with a circulating fluidized bed and vertical pneumatic transport which have an effect on the hydrodynamics of the two-phase medium.

In conclusion, we note that the proposed diagram of states of a disperse medium with ascending gas flow includes all the currently known modifications of such systems and contains recommendations for calculation of the pressure gradients and of all, in practice, boundary velocities of the gas. For the first time a simple expression has been obtained to evaluate the most important characteristic of a flow system, i.e., the transport velocity (22). This makes the phase diagram proposed useful for practical application in engineering practice.

NOTATION

A, dimensionless parameter in (25); $B = \left(\frac{\rho_s}{\rho_f}\right)^{1/2} \frac{\sqrt{J_s^*}}{\mathrm{Fr}_t}$; d, diameter of the particle; D, diameter of the riser; D_b and D_h, frontal and vertical diameters (dimensions) of the gas bubble; $\mathrm{Fr}_h = \frac{(u - u_{mf})^2}{gh}$, $\mathrm{Fr}_H = \frac{u_t^2}{gH}$, and $\mathrm{Fr}_t = \frac{(u - u_t)^2}{gH}$, Froude numbers; g, free-fall acceleration; H, height of the bed (riser); H_{mf}, height of the fluidized bed for $u = u_{mf}$; h, height above the gas distributor; J_s, specific mass circulation flux of particles; $\bar{J}_s = J_s/\rho_s(u - u_t)$, $\tilde{J}_s^* = J_s/\rho_s u_t$, and $J_s^* = J_s/\rho_f u_t$, dimensionless mass fluxes of particles; L, vertical dimension of the cluster; Δp, pressure difference over the height Δh; u, filtration rate of the gas; z, height of the fluidized bed at the grid in the circulating fluidized bed; ε, porosity of the bed; μ_f, dynamic viscosity of the gas; ρ, density. Subscripts: b, gas bubble; f, gas; mf, beginning of fluidization; s, particles; t, free-fall conditions of a single particle; w, at the riser wall; 0, conditions of a fixed bed; fr, friction; opt, optimum; sl, piston regime; tr, transport; int.c, internal circulation; d, descent.

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